PLASMA FLOW ACCOMPANIED BY BLOWING AND PUMPING OF PLASMA ACROSS ELECTRODES

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The article discusses plane stationary slowly varying flows of a nonviscous plasma with good conductivity in a channel in a transverse magnetic field; the flows are accompanied by blowing in and pumping plasma across solid metallic electrodes. The Hall effect is taken into consideration. It is shown that the potential jump near the anode, which appears in an accelerated plasma flow in an ordinary channel with solid electrodes, can be eliminated in flows accompanied by blowing in (pumping) of plasma. It is also shown that flows are possible in which the velocity, density, and the transverse electric field increase in the direction of the accelerator cathode.

It is well known from experiments conducted with large-current* stationary coaxial plasma accelerators with solid electrodes [1, 2] that the acceleration of plasma is accompanied by the development of a longitudinal electric current, which pushes the plasma toward the cathode of the accelerator, and a large potential jump near the anode. The longitudinal electric current drawn far beyond the accelerator has a tendency to form current ties at the anode both in the accelerator channel itself and in its exit section. The thermal overloading of the anode arising as a result of this reduces the stability of the entire system sharply and does not permit an increase of the discharge current when a high velocity of the outflow is desired. Besides, an increase of the discharge current also leads to the loss of stability of the flow.

These phenomena are accounted for by the strong influence of the Hall effect [3-6]. As is well known [7], in an ideally conducting plasma the electric potential is conserved along electron trajectories. Since it is just the electrons that carry electric current, in a good-conducting plasma the equipotentials of the electric field are pressed to the anode of the system as the plasma gets accelerated.

One of the conceivable ways of eliminating large potential jumps near the anode and the current ties is to obtain flow regimes with the feed of the plasma across the electrodes, in particular the anode. In this case, on the one hand the density of the plasma near the anode increases, which increases the possibility of the electrons hitting the anode when they approach it. On the other hand, some fraction of the discharge current is carried by ions, and hence, a more uniform distribution of the discharge current along the surface of the electrodes is possible.

The flows in MHD-generator channels associated with the feed and pumping across the electrodes have been investigated in [8], where it is shown that the nonuniform current distribution along the electrodes caused by the Hall effect [9] becomes more uniform. The object of the present work is to investigate plane stationary plasma flows accompanied by the feed and pumping of plasma across electrodes in the channel of a large-current accelerator. The Hall effect is taken into consideration. The investigation is carried out under the assumption of slowly varying flows.

1. Initial System of Equations. We consider a plane flow of fully ionized quasineutral plasma in a channel in an intrinsic (i.e., produced by the discharge current) transverse magnetic field (Fig. 1).

*The characteristic parameters of a large-current accelerator are $v \sim 10^7$-10^8 cm/sec, number of particles in 1 cm^3 $n \sim 10^{14}$-10^{15}, temperature $T \approx 2$ eV, discharge current $I \sim 10^3$-10^4 A.

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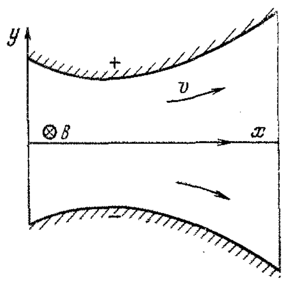

Fig. 1

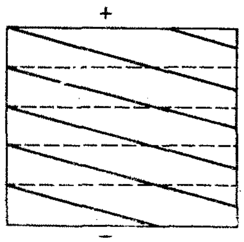

Fig. 2

The walls of the channel are the electrodes. All the flow parameters are assumed to be functions of the variables x, y. The magnetic field **B** is oriented along the z axis; in this direction the channel is assumed to be infinitely wide. The velocity vector **v**, the electric-field vector **E**, and the electric-current-density vector **j** lie in the plane of the flow, i.e., in the xy plane. The axis x is directed along the channel. We shall assume that the plasma is nonviscous and does not conduct heat, that the ions are singly charged, that the inertia of the electrons is negligibly small, and that the equations of state of the plasma components (electrons and ions) are given by polytropic dependence $p_k = p_k(\rho)$. In this case the equations describing the stationary flow have the form

$$\begin{gathered}\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla P, \quad \operatorname{div}\rho\mathbf{v} = 0, \quad \mathbf{E} = -\nabla\varphi \\ P = p(\rho) + \frac{B^2}{8\pi}, \quad \frac{\mathbf{j}}{\sigma} = \mathbf{E}_T + \frac{\mathbf{v}}{c}\times\mathbf{B} + \frac{M}{e\rho}\nabla P \\ \operatorname{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \quad \mathbf{E}_T = -\nabla\left(\varphi + \frac{M}{e}\int\frac{dp_i(\rho)}{\rho}\right) = -\nabla\varphi_T \\ p_i = p_i(\rho), \; p_e = p_e(\rho)\end{gathered} \tag{1.1}$$

We shall specify the mass $m^{\cdot}$ of the matter (plasma) flowing per second across the input section of the channel. We choose the variables s, ψ, in which

$$\rho v_x = m^{\cdot}\frac{\partial\psi}{\partial y}, \qquad \rho v_y = -m^{\cdot}\frac{\partial\psi}{\partial x} \tag{1.2}$$

instead of the variables x, y to describe the flow; here ψ is the stream function of the plasma and s is the length of the curve ψ =const. Introducing the radius of curvature r of the curves ψ =const, the first and the fifth equations in (1.1) can be written in the form

$$\begin{gathered}\rho v\frac{\partial v}{\partial s} = -\frac{\partial P}{\partial s}, \qquad \frac{\rho v^2}{r} = -\frac{\rho v}{m^{\cdot}}\frac{\partial P}{\partial\psi} \\ \frac{\nu_m}{c}\frac{\rho v}{m^{\cdot}}\frac{\partial B}{\partial\psi} = -\frac{\partial\varphi_T}{\partial s} + \frac{M}{e\rho}\frac{\partial P}{\partial s} \qquad \left(\nu_m = \frac{c^2}{4\pi\sigma}\right) \\ -\frac{\nu_m}{c}\frac{\partial B}{\partial s} = -\frac{\rho v}{m^{\cdot}}\frac{\partial\varphi_T}{\partial\psi} - \frac{vB}{c} + \frac{M}{e\rho}\frac{\rho v}{m^{\cdot}}\frac{\partial P}{\partial\psi}\end{gathered} \tag{1.3}$$

From the second equation in (1.3) we get

$$\frac{vB/c}{(Mv/em^{\cdot})\,\partial P/\partial\psi} = \frac{r}{\Lambda} \qquad \left(\Lambda = \frac{Mcv}{eB}\right) \tag{1.4}$$

We denote by L the characteristic longitudinal (along the line ψ =const) scale of variation of the flow parameters:

$$\frac{\partial(\cdot)}{\partial s} \sim \frac{1}{L}(\cdot) \tag{1.5}$$

The flow will be called slowly varying if the conditions

$$\frac{L}{r} \ll 1, \qquad \frac{\Lambda}{r} \ll 1 \tag{1.6}$$

are satisfied.

Furthermore, we shall assume that the magnetic Reynolds number is large compared to unity:

$$R_m = \frac{v_0 L}{\nu_m} \gg 1 \tag{1.7}$$

Here v_0 is the characteristic velocity of the plasma.

When conditions (1.6) and (1.7) are satisfied, in the first approximation the second equation in (1.3) can be discarded, and the terms $(\nu_m/c)\partial B/\partial s$ and $(Mv/em^{\cdot})\,\partial P/\partial\psi$ in the fourth equation in (1.3) can be neglected. As a result we obtain the following system of equations:

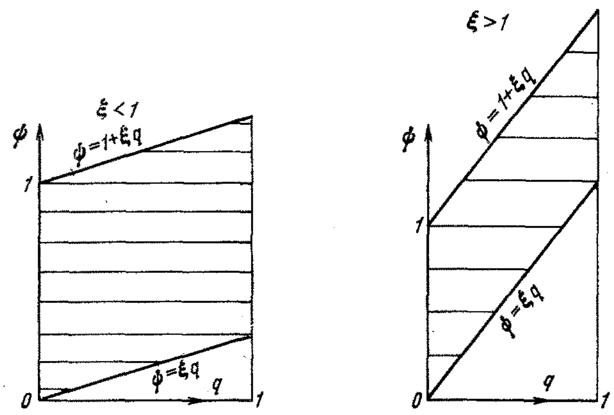

Fig. 3

$$\rho v \frac{\partial v}{\partial s} = -\frac{dP}{ds}, \qquad P(s) = p(\rho) + \frac{B^2}{8\pi}$$
$$\frac{\nu_m}{c}\frac{\rho v}{m^*}\frac{\partial B}{\partial \psi} = -\frac{\partial \varphi_T}{\partial s} + \frac{M}{e\rho}\frac{dP}{ds} \tag{1.8}$$
$$\frac{B}{\rho c} = -\frac{1}{m^*}\frac{\partial \varphi_T}{\partial \psi}, \qquad \varphi_T = \varphi + \frac{M}{e}\int \frac{dp_i}{\rho}$$

Conditions (1.6) signify that the radius of curvature of the streamlines ψ = const must be sufficiently large. If the flow is not accompanied by feed and pumping of the plasma across the electrodes, i.e., the electrodes (the channel walls) themselves are streamlines ψ = const, then in this case the transverse cross section of the channel must change sufficiently slowly. Noting that for such a flow $r \sim (d^2f/dx^2)^{-1} \sim L/(df/dx)$, where f is the width of the transverse cross section of the channel, from (1.6) we have

$$\left|\frac{df}{dx}\right| \ll 1 \tag{1.9}$$

and the approximation of slowly varying flow is equivalent to the approximation of a channel with slowly varying cross section. Noting also that

$$\frac{\partial(\cdot)}{\partial s}\Big|_{\psi=\mathrm{const}} \approx \frac{\partial(\cdot)}{\partial x}\Big|_{\psi=\mathrm{const}}$$

where the x axis is directed along the channel, from (1.8) we obtain equations describing plane flow in a channel with slowly varying cross section, derived earlier in [6].

The results of the computation of the flow with the use of Eqs. (1.8) can be refined by substituting the obtained results into the second equation and retaining the discarded terms of the fourth equation of the exact system of equations (1.3).

If the electrodes are solid metallic electrodes, $p \sim \rho^\gamma$ and $\beta = 8\pi p/B^2 \ll 1$, and the system of equations (1.8) can be simplified further. In this case we find (see [6]) that

$$\rho = \frac{\rho_0(1-q)}{\partial\Phi/\partial\psi}, \qquad \rho_0 = \frac{4\pi e m^{*2}\xi}{Mc^2U}$$
$$B = -U\frac{\rho_0 c(1-q)}{m^*}\left[1 - \frac{p(\rho)}{2P(s_0)(1-q)^2}\right] \tag{1.10}$$
$$q(s) = 1 - \left(\frac{P(s)}{P(s_0)}\right)^{1/2}, \qquad \Phi = \frac{\varphi}{U}, \qquad u = v\left(\frac{M}{2eU\xi}\right)^{1/2}$$

The dimensionless velocity u and potential Φ satisfy the equations

$$u\frac{\partial^2\Phi}{\partial\psi^2} = \frac{c^2U^2}{4\pi\nu_m\gamma p_0}\left(\frac{M}{2eU\xi}\right)^{1/2}\frac{1}{(1-q)^\gamma}\frac{dq}{ds}\left(\frac{\partial\Phi}{\partial\psi}\right)^{\gamma+2}\left(\frac{\partial\Phi}{\partial q} + \xi\frac{\partial\Phi}{\partial\psi}\right)$$
$$\frac{\partial u^2}{\partial q} = \frac{\partial\Phi}{\partial\psi} \qquad \left(\xi = \frac{Mc\,|B(s_0)|}{4\pi e m^*}, \quad p = p_0\left(\frac{\rho}{\rho_0}\right)^\gamma\right) \tag{1.11}$$

Here U is the potential difference between the electrodes, ξ is the exchange parameter for an infinitely long channel (we assume that the magnetic field tends to zero at the exit from the channel), and p_0 is the characteristic pressure, and also the subscript 0 denotes the value of the quantities at the entrance to the channel. Henceforth we shall assume that $s_0 = 0$.

For flows accompanied by feed and pumping of the plasma across the electrodes ψ is not constant at the walls of the channel. Denoting the value of ψ at the anode by $\psi_a(q)$ and at the cathode by $\psi_k(q)$, we obtain the boundary conditions for the dimensionless potential:

$$\Phi(q, \psi_a) = 1, \qquad \Phi(q, \psi_k) = 0 \tag{1.12}$$

For the chosen normalization at the entrance to the channel, i.e., for q=0, we have $\psi_a(0)=1$ if $\psi_k(0)$ is chosen equal to zero.

We shall investigate three characteristic regimes of flow with the feed of the plasma across the electrodes. It is assumed that the plasma fed across the electrodes has the same characteristics as the plasma in the main flow.

2. Flow Accompanied by Feed of Plasma across Anode and Pumping across Cathode. We consider a flow regime in which the potential Φ and the velocity u are given by the relations

$$\Phi=\psi-\xi q, \qquad u=\sqrt{q+g(\psi)} \tag{2.1}$$

and

$$B=B_0(1-q), \quad \rho=\rho_0(1-q)$$
$$B_0=-cU\rho_0/m^{\cdot}$$

We introduce an electron stream function ψ_e by a relation similar to (1.2):

$$\mathbf{v}_e=\mathbf{v}-\frac{M\mathbf{j}}{e\rho}, \qquad \rho\mathbf{v}_e=m^{\cdot}\nabla\psi_e\times\mathbf{n}_z \tag{2.2}$$

Here $\mathbf{n}_z$ is the unit vector in the direction of the z axis.

It is easy to see that the potential Φ defined by (2.1) will be constant along electron trajectories ψ_e =const (dashed lines in Fig. 2). Hence, in the investigated regime the electrical current is carried by ions (continuous lines in Fig. 2).

From the boundary conditions (1.12) we obtain

$$\psi_k(q)=\xi q, \qquad \psi_a(q)=1+\xi q \tag{2.3}$$

Thus the total mass $m_a^{\cdot}$ of the plasma fed across the anode and pumped out across the cathode is equal to

$$m_a^{\cdot}=\xi m^{\cdot} \tag{2.4}$$

It is obvious (Fig. 3a, b) that for $\xi<1$ the ions starting from the entrance of the channel reach the exit, while for $\xi>1$ only ions fed across the anode reach the exit.

The width of the channel $f(q)$* is determined from the expression

$$f(q)=m^{\cdot}\int_{\psi_k}^{\psi_a}\frac{d\chi}{\rho(q,\chi)\,v(q,\chi)}=\frac{m^{\cdot}}{\rho_0 v_0(1-q)}\int_{\xi q}^{1+\xi q}\frac{d\chi}{\sqrt{q+g(\chi)}}\quad\left(v_0=\left(\frac{2eU\xi}{M}\right)^{1/2}\right) \tag{2.5}$$

The longitudinal (in the direction of the velocity v of the ions) electric field E_s is

$$E_s=-U\frac{\partial\Phi}{\partial q}\frac{dq}{ds}=\xi U\frac{dq}{ds} \tag{2.6}$$

The transverse electric field E_n is determined from the relation

$$E_n=-U\frac{\rho v}{m^{\cdot}}\frac{\partial\Phi}{\partial\psi}=-U\frac{\rho_0(1-q)v_0}{m^{\cdot}}[q+g(\psi)]^{1/2} \tag{2.7}$$

In particular, the electric field at the anode is

$$E_a=-\left\{\left(\xi U\frac{dq}{ds}\right)^2+\left[U\frac{\rho_0 v_0(1-q)}{m^{\cdot}}\right]^2[q+g(1+\xi q)]\right\}^{1/2} \tag{2.8}$$

Thus, if the velocity of the plasma near the anode, determined by the feed conditions, is not too large, then the electric field also is not large. Hence it follows that by a suitable choice of the rate of plasma

*Actually $f(q)$ represents the arc length of the curve orthogonal to the streamline ψ = const. However, in the investigated approximation, when the radius of curvature r of the lines ψ =const is large ($r\to\infty$), the difference between f and the true width of the channel is small if the axis of the channel is so chosen that the angle between it and the streamlines ψ =const is sufficiently small.

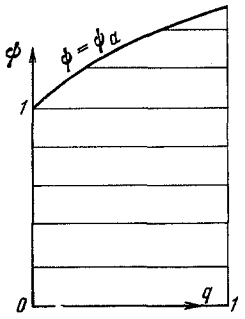

Fig. 4

feed across the anode it is possible to eliminate the potential jump near the anode, which is so characteristic of the flow without plasma feed across the anode; what is more, the potential drop in the region near the anode can be regulated within very wide ranges.

As a specific example we consider the flow in which the rate of plasma feed across the anode is small. Thus we choose the function $g(\psi)$ in the form

$$g(\psi) = \begin{cases} 0 & (0 \leqslant \psi \leqslant 1) \\ \xi^{-1}(\psi - 1)(\theta^2 - 1) & (1 \leqslant \psi \leqslant 1 + \xi q) \end{cases} \tag{2.9}$$

where $\theta = \text{const}$. In this case

$$u = \begin{cases} \sqrt{q} & (0 \leqslant \psi \leqslant 1) \\ [q + \xi^{-1}(\psi - 1)(\theta^2 - 1)]^{1/2} & (1 \leqslant \psi \leqslant 1 + \xi q) \end{cases} \tag{2.10}$$

At the anode $u = \theta\sqrt{q}$ $(\theta > 0)$. We shall assume that the constant θ is small in comparison with unity:

$$\theta \ll 1 \tag{2.11}$$

Then we obtain

$$f(q) \approx \frac{m^\cdot (1 + \xi q)}{\rho_0 v_0 (1 - q)\sqrt{q}} \tag{2.12}$$

$$E_a = -\left\{\left(\xi U \frac{dq}{ds}\right)^2 + \theta^2 q \left[U \frac{\rho_0 v_0 (1 - q)}{m^\cdot}\right]^2\right\}^{1/2} \tag{2.13}$$

The reactive thrust F is determined by the expression (it is assumed that the plasma is accelerated to the value $q = 1$)

$$F = \int_{\xi}^{1+\xi} m^\cdot v_0 u \, d\psi \tag{2.14}$$

For $\xi < 1$ and $\xi > 1$ respectively we have

$$F = m^\cdot v_0 \left(1 - \frac{\xi}{3}\right), \qquad F = \frac{2}{3}\frac{m^\cdot v_0}{\sqrt{\xi}} = \frac{2}{3} m^\cdot \left(\frac{2eU}{M}\right)^{1/2} \tag{2.15}$$

Thus the thrust is independent of ξ for $\xi > 1$. We compute the energy Q removed per second along with the plasma drawn across the cathode. Obviously

$$Q = \frac{m^\cdot}{M} \int_0^{\xi} \frac{M}{2} v_0^2 u^2 \, d\psi \tag{2.16}$$

Hence

$$Q = \begin{cases} {}^1\!/_2 N\xi & (\xi < 1) \\ N(1 - {}^1\!/_2 \xi^{-1}) & (\xi > 1) \end{cases} \qquad \left(N = \frac{eUm^\cdot \xi}{M}\right)$$

Here N is the electrical power supplied to the accelerator.

The energy W spent in accelerating the plasma is*

$$W = N - Q = \begin{cases} N(1 - {}^1\!/_2 \xi) & (\xi < 1) \\ {}^1\!/_2 N\xi^{-1} = {}^1\!/_2 eUm^\cdot M^{-1} & (\xi > 1) \end{cases} \tag{2.17}$$

*The thermal energy of the plasma was not taken into consideration in the computation. A correct consideration of the effect of temperature also requires the consideration of the difference between Φ_T and Φ. However, the thermal corrections are of the order of β and are small for $\beta \to 0$.

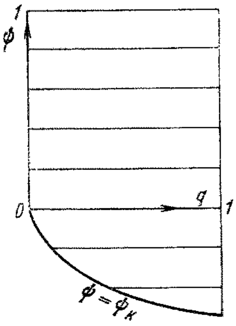

Fig. 5

Specifying the density ρ_0 at the entrance to the channel, with the use of (1.10) we obtain the volt-ampere characteristic:

$$U = \frac{4\pi I m^*}{c^2 \rho_0} \qquad \left(\xi = \frac{MI}{em^*}\right) \tag{2.18}$$

The magnetic field B_0 at the entrance to the channel is equal to $-4\pi I/c$; therefore for v_0 we obtain

$$v_0 = \frac{|B_0|}{\sqrt{2\pi\rho_0}} \tag{2.19}$$

We stress that the dependence q(s) for the investigated regime is quite arbitrary; it is only necessary that the condition $dq/ds > 0$ is satisfied (the regime of acceleration).

In conclusion we note that if small perturbations appear in the flow, then the equation for the perturbation of the potential has the same structure as the equation for the perturbation of the isomagneticity parameter (B/ρ), investigated earlier in [3]. However, it does not follow that for highly pronounced Hall effect (large ξ) the presence of perturbations will change the flow pattern drastically, since the unperturbed electron trajectories, along which the potential perturbations propagate, do not intersect the electrodes, i.e., the walls of the channel. Therefore, even in the case of large ξ an electromagnetic layer is not formed near the anode in the presence of perturbations.

3. Flow with Plasma Feed across Anode. We now consider the flow regime in which the plasma is fed across the anode but is not pumped out across the cathode (Fig. 4). Flows of this type can be realized in the accelerator part of a magnetoplasma compressor (MPC). In this regime ψ (q) = 0. Let the conductivity of the plasma σ be constant for the sake of definiteness. We choose the coefficient A (q) in Eq. (1.11) for Φ in the form

$$\xi^{\gamma+2} A(q) \frac{dh(\xi q)}{dq} = G_0 = \text{const} \tag{3.1}$$

We choose the potential Φ and the velocity u in the form

$$\Phi = \xi\,[h(\xi q) - h(\xi q - \psi)], \qquad u^2 = h(\xi q - \psi) \tag{3.2}$$

Then for the function h we obtain the equation

$$\sqrt{h(\zeta)}\, h'' = -G_0 (h')^{\gamma+2} \qquad (\zeta = \xi q - \psi) \tag{3.3}$$

The prime denotes differentiation of the function h with respect to ζ.

Integrating Eq. (3.3), we get

$$\frac{\zeta}{(2\gamma G_0)^{1/\gamma}} + C_1 = 2\gamma \left[\frac{(\sqrt{h}+C_2)^{2+1/\gamma}}{2\gamma+1} - C_2 \frac{(\sqrt{h}+C_2)^{1+1/\gamma}}{\gamma+1}\right] \tag{3.4}$$

where $C_{1,2}$ are constants.

At the entrance to the channel (q = 0) we have $\Phi(0,0) = 0$, $\Phi(0,1) = 1$, and thus

$$h(-1) = h(0) - 1/\xi \tag{3.5}$$

Hence it follows that the velocity of the plasma at the anode is smaller than at the cathode. Assuming that $h(-1) = 0$, i.e., the plasma velocity at the anode at the entrance to the channel is equal to zero, we have

$$h(0) = 1/\xi \tag{3.6}$$

For simplicity we choose the constant C_2 equal to zero, so that

$$G_0 = \xi^{\gamma+1/2} (2\gamma+1)^{\gamma} (2\gamma)^{-(\gamma+1)}$$

then

$$u^2 = h(\xi q - \psi) = \xi^{-1}(\xi q + 1 - \psi)^{\varkappa}$$

$$\Phi = (\xi q + 1)^{\varkappa} - (\xi q + 1 - \psi)^{\varkappa} \qquad \left(\varkappa = \frac{2\gamma}{2\gamma + 1}\right) \tag{3.7}$$

The function $\psi_a(q)$ is found from the condition $\Phi(q_1, \psi_a) = 1$.

Thus we obtain

$$\psi_a(q) = 1 + \xi q - [(1 + \xi q)^{\varkappa} - 1]^{1/\varkappa} \tag{3.8}$$

As was to be expected $\psi_a(q) \geq 1$. It follows from (3.8) that $\psi_a \to 1$ for $\xi \to 0$. If $\xi \to \infty$, then for $\xi q \gg 1$ we have

$$\psi_a \approx \varkappa^{-1}(\xi q)^{1-\varkappa}$$

Thus, for the limiting cases $\xi \to 0$ and $\xi q \gg 1$ the mass $m_a^{\cdot}$ of the plasma fed across the anode is respectively equal to

$$m_a^{\cdot} = \begin{cases} \xi m^{\cdot} \\ m^{\cdot}\varkappa^{-1}\xi^{1-\varkappa} \end{cases}$$

The velocity v and the density ρ are respectively equal to

$$v = \left(\frac{2eU}{M}\right)^{1/2}(1 + \xi q - \psi)^{\varkappa/2}, \qquad \rho = \rho_0\varkappa^{-1}(1 - q)(1 + \xi q - \psi)^{1-\varkappa} \tag{3.9}$$

It is evident that the density of the plasma near the cathode is higher than near the anode. In particular, $\rho \to 0$ near the anode at the entrance to the channel.

The reactive thrust F is

$$F = \frac{2}{2 + \varkappa} m^{\cdot}\left(\frac{2eU}{M}\right)^{1/2}\left\{(1 + \xi)^{1-\varkappa/2} - [(1 + \xi)^{-(1/\varkappa+1/2\gamma)} - 1]^{1/\varkappa+1/2\gamma}\right\} \tag{3.10}$$

The longitudinal and transverse components of the electric field are respectively equal to

$$E_s = -\varkappa\xi U \frac{dq}{ds}[(1 + \xi q)^{\varkappa-1} - (1 + \xi q - \psi)^{\varkappa-1}]$$

$$E_n = -U\rho_0\frac{(1 - q)}{m^{\cdot}}\left(\frac{2eU}{M}\right)^{1/2}(1 + \xi q - \psi)^{\varkappa/2} \tag{3.11}$$

In particular, at the cathode

$$E_n = -U\rho_0\frac{(1 - q)}{m^{\cdot}}\left(\frac{2eU}{M}\right)^{1/2}(1 + \xi q)^{\varkappa/2}, \qquad E_s = 0 \tag{3.12}$$

while at the anode

$$E_n = -U\rho_0\frac{(1 - q)}{m^{\cdot}}\left(\frac{2eU}{M}\right)^{1/2}[(1 + \xi q)^{\varkappa} - 1]^{1/2}$$

$$E_s = -U\frac{dq}{ds}\varkappa\xi\{(1 + \xi q)^{\varkappa-1} - [(1 + \xi q)^{\varkappa} - 1]^{1/2\gamma}\} \tag{3.13}$$

It is evident that the transverse electric field E_n decreases in absolute value in the direction of the anode. In the regime under consideration also there is no potential jump near the anode.

The width of the channel f is found to be equal to

$$f = \frac{2m^{\cdot}}{\rho_0(2eU/M)(1 - q)}\{(1 + \xi q)^{\varkappa/2} - [(1 + \xi q)^{\varkappa} - 1]^{1/2}\} \tag{3.14}$$

The dependence q(s) is determined by the expression

$$\int_0^q \frac{(1 + \xi\mu)^{\varkappa-1}}{(1 - \mu)^{\gamma}}d\mu = s\frac{4\pi v_m\gamma p_0}{c^2U^2\xi}\left(\frac{2eU}{M}\right)^{1/2}\frac{(2\gamma + 1)^{\gamma+1}}{(2\gamma)^{\gamma+2}} \tag{3.15}$$

The integral on the left side of (3.15) can be expressed in terms of elementary functions in a complicated way only for $\gamma = 1$ and $\gamma = 2$. However, it is evident that $s \sim q$ for $q \to 0$, and for $q \to 1$ we have $s \sim (1-q)^{1-\gamma}$ $(\gamma > 1)$.

To sum up it can be said that an increase of the density, velocity, and electric field (in absolute value) in the direction of the cathode is characteristic for the investigated regime.

4. Flow with Plasma Feed across Cathode. We now consider briefly the flow regime accompanied by plasma feed across the cathode (Fig. 5). In this case $\psi_a(q) = 1$. We put

$$A(q)\,\xi^{\gamma+2}\,dh(\xi q)\,/\,dq = G_0 = \text{const}$$

We shall seek the solution of Eqs. (1.11) in the form

$$u^2 = h(\xi q + 1 - \psi),\quad \Phi = 1 + \xi\,[h(\xi q) - h(\xi q + 1 - \psi)] \tag{4.1}$$

Then for $G_0 = (2\gamma + 1)^{\gamma}\,\xi^{\gamma+1/2}\,(2\gamma)^{-(\gamma+1)}$ and under the condition $h(0) = 0$ we have

$$h(\xi q + 1 - \psi) = \xi^{-1}(\xi q + 1 - \psi)^{\varkappa} \qquad \left(\varkappa = \frac{2\gamma}{2\gamma + 1}\right) \tag{4.2}$$

Thus, formally the solution has the same form (and, hence, the same properties) as that investigated in the preceding section. In particular, the density, velocity, and the electric field increase in the direction of the cathode in this case also.

The function $\psi_k(q)$ is equal to

$$\psi_k(q) = 1 + \xi q - [1 + (\xi q)^{\varkappa}]^{1/\varkappa} \tag{4.3}$$

It is easy to see that $\psi_k(q) \leq 0$, i.e., the plasma is actually blown into the cathode. However, the rate of blowing in is too large for this accelerator to be suitable energetically.

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